Experiments on two-dimensional flow over a normal wall

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SUMMARY

Measurements of the velocity, pressure, and turbulence behind a series of normal plates in the uniform test section of an air tunnel are described, the oscillation of the wake being prevented in all but one test through use of symmetrically located tail plates. By a combination of experimental and computational techniques, details of the pattern of flow over a wall on a plane boundary in an infinite fluid are closely approximated. A significant difference is indicated between the characteristics of such a flow and those of the flow past an isolated plate with oscillating wake.

INTRODUCTION

For want of experimental information about the characteristics of flow past a sufficiently great variety of boundary shapes, one is forced either to perform specific tests upon each body design that comes into question, or to estimate the expected characteristics from what is known about flow past the individual elements of which the body consists. Under some conditions the latter method is quite effective; often the purely qualitative indication that it provides is far better than none at all, and sometimes the combination of available data with mathematical analysis permits the indication to be of quantitative value as well. Under other conditions the information at hand is merely sufficient to throw serious doubt upon even the qualitative value of such a procedure.

A case in point is the utilization of experimental data for two-dimensional flow past a cylindrical body to predict the conditions of motion that would prevail if a boundary were to exist at the plane of symmetry. Assume, for example, that it was desired to estimate the force exerted by the wind upon a wall, or the effectiveness of such a wall in either shielding something behind it or in promoting the diffusion of some substance in its wake. Drag coefficients and eddy patterns are well known for rectangular plates (Fage & Johansen 1927, Goldstein 1938) but it is also known (Roshko 1955) that interference with an oscillating wake is likely to change the primary nature of the flow. Just how great this change might be in any particular case can be determined as yet only by experiment.

Whereas the wake behind an immersed body is of basic importance in such professions as aeronautics, ballistics, and marine engineering, it is the wake behind a boundary irregularity, such as the wall, that is of primary interest in hydraulics. Whether the wall typifies roughness elements or part of an orifice or valve makes little difference; and the standing eddy in its wake also has much in common with those which form at any abrupt changes in conduit section. Yet so little is known about their geometrical proportions, about the dynamics of the mean flow within them, and about the generation of turbulence along their borders, that the experimental analysis of such standing eddies has received the attention of the Iowa Institute of Hydraulic Research for nearly a decade. The most recent in this series of investigations has been a study of the wake behind a simple wall (Arie 1955), the purpose of the present paper being to make generally available the most essential of the results.

Flow past a plate that is fully surrounded by fluid differs in several ways from flow past one that is in contact with a continuous boundary. First of all, the former may be considered to lie in the path of essentially irrotational flow, whereas the latter will be immersed in a boundary layer of indefinite thickness. Secondly, as has already been noted, the flow in the wake of an isolated plate is free to oscillate about the plane of symmetry, whereas such oscillation is wholly prevented in the vicinity of a rigid longitudinal boundary. Now the lateral freedom or restraint of the wake marks a dynamically basic distinction between the two cases, whereas the boundary layer is a secondary factor that is not determined by local conditions. Hence, in the experiments under discussion, boundary layer development both upstream and downstream was obviated by placing the plate at the axis, rather than at the floor, of an air tunnel, the lateral boundary restraint at the rear being effected by means of a tail plate which was only slightly longer than the standing eddy.

Whether a plate fully surrounded by the fluid or one in contact with a boundary is simulated in a tunnel, the presence of the test section floor and ceiling will introduce an additional threefold effect. First, these surfaces will correspond geometrically to planes of symmetry between the given plate and standing eddy, on the one hand, and their first images in a series having a finite lateral spacing, on the other; the velocity of the surrounding field being increased thereby. Second, there will be an additional augmentation of velocity as a result of the development of the boundary layer along the tunnel surfaces. Finally, despite the use of a foreshortened downstream boundary, the conversion of energy in the wake of the test plate will produce something akin to a boundary layer effect, which will cause the velocity of the flow outside the wake to become still higher than it would be in an infinite fluid.

Three courses were thus open to the investigator. The pattern of flow could be determined as a function of the relative spacing of a series of plates, with the double aim of permitting extrapolation to the infinite case and of providing supplementary information about flow between opposite walls in a passage of finite size. Secondly, by means of various experimental artifices, an effort could be made to simulate the infinite case itself without recourse to extrapolation. Finally, the data obtained for any spacing could be recomputed for infinite spacing by means of an analytical approximation. In actuality, these three courses were followed in sequence, the first serving as a guide to the partial refinements of the second, and the final correction proceeding analytically.

Experimental procedure

The air tunnel used for the experiments in question was of the closed circuit type, with a uniform test section 3 feet high, 3 feet wide, and 10 feet long. Air speeds could be varied at will from 10 to 50 feet per second, but for ease in measurement all tests were made in the upper part oı this range. The main air speed was indicated by the change in pressure at the bell entrance to the test section; the corresponding pressure-velocity relationship had been determined through test-section traverses with a Prandtl-type Pitot tube, which in turn had been calibrated in the irrotational core of a jet from a 6-inch rounded orifice. No effort was made to reduce the turbulence of the flow, beyond the use of vanes at the turns and a fourfold reduction in cross-sectional area between the plenum chamber and the test section; the root-mean-square velocity fluctuation was about $1\frac{1}{2}\%$ of the mean velocity of flow.

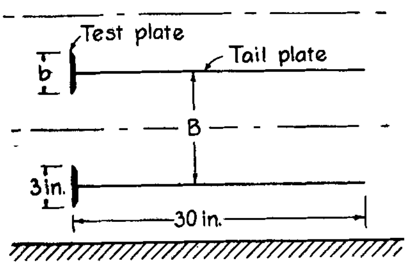

Figure 1. Definition sketch of plate arrangement.

All test plates were introduced into the tunnel approximately midway between the ends of the test section, completely spanning its 3-foot width. They were invariably 3 inches in effective height, 1/4 inch in thickness, and bevelled at an angle of 60° along the edges on the rear side. All but one were of tempered hardboard, the one being of carefully machined brass. Each was guyed with piano wire, at its one-third points, to the floor and ceiling of the tunnel to maintain its alignment and eliminate all possibility of vibration. The tail plates (see figure 1), which were of 1/16-inch sheet aluminium, were attached to the test plates at mid-height, and likewise guyed in place. They extended a distance of 30 inches to the rear, which had been found by observation to be longer than the eddy by a sufficient amount for the size of the latter to be unaffected by a further lengthening of of the plate. By successively introducing dummy plate assemblies at planes of symmetry on either side of the centrally located test plate, the ratio B/b of spacing to width could be varied through the consecutive values of 12, 6, 3, etc.

Because of the two-dimensional nature of the flow, it was possible to make measurements of the magnitude and direction of the mean velocity at any point by means of one of two alternative Pitot cylinders. Each was constructed of 1/8-inch brass tubing mounted horizontally on a fork-shaped support with bearings 6 inches on centre. One cylinder contained a 0·015-inch hole on each side of a central plug (see figure 2), the two portions

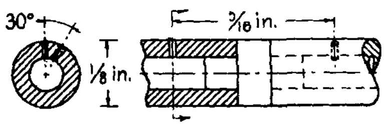

Figure 2. Pitot-cylinder details.

of the tube having been rotated differentially about their common axis till the holes lay at an angle of 30° to each other in the normal plane. A protractor at either bearing permitted the common angular position of the holes to be read to within about 0·2°. The other cylinder differed from the first only in the angle—now 180°—between the holes. After careful calibration of each cylinder, the magnitude of the velocity could be computed from the differential pressure between the respective holes, the 180° cylinder being oriented to yield the maximum reading, and the 30° cylinder being turned so that first one and then the other hole lay directly upstream (in the latter instance the direction of the flow was first determined by noting the angle at which the differential reading was exactly zero). The 30° cylinder was used so long as the velocity gradient was not excessive; since, in zones of high gradient, this cylinder was subject to a pronounced error in both direction and magnitude of the velocity, the 180° cylinder was then employed, and the direction determined by successive steps in plotting the streamlines.

In the zones in which both the inclination and the magnitude of the velocity vector were directly measurable, it was a simple matter to determine and plot, from vertical traverses with the 30° cylinder, the distribution of the longitudinal component of velocity over a series of normal sections. Graphical calculations of the areas $\psi = \int u\,dy$ enclosed by these curves (figure 3), as a function of distance normal to the tail plate, then permitted successive streamlines $\psi = C$ to be located for any selected increment $\Delta\psi$. In those zones for which only the magnitude of the velocity could be measured, this was first assumed to be indicative of the longitudinal component. Evaluation of the streamline location according to the foregoing method yielded the inclination of the velocity vectors as a first approximation, and a second approximation was usually sufficient to determine the flow pattern with acceptable precision. Figure 4 shows the pattern obtained from the single run made without tail plate for purposes of reference.

This method of determining the flow pattern was followed for B/b equal to 12 and to 6 without undue experimental difficulty. All attempts to repeat the measurements for B/b equal to 3 met with no success, however,

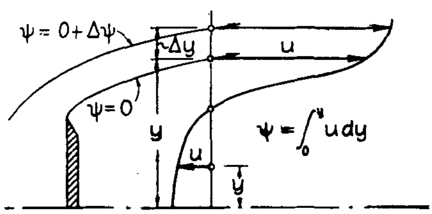

Figure 3. Determination of streamline location.

as the pattern was then not only unsymmetrical about the axis of any plate but unstable as well. This situation is in approximate accord with previous experience, flow through lattice screens being known to become unstable as the solidity ratio approaches the magnitude 0·5. For this reason, no attempt was made to carry the study any farther in the direction of decreasing spacing, which meant that a sufficient number of cases to

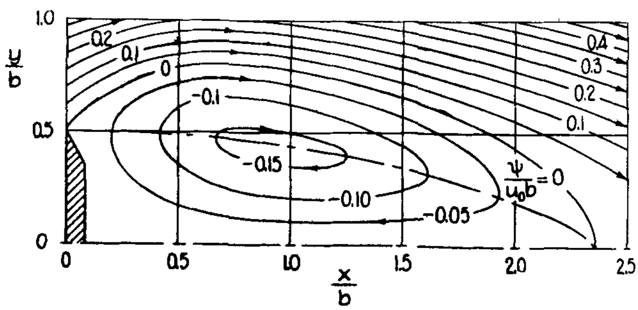

Figure 4. Approximate pattern of mean streamlines for flow without tail plate $B/b = 12$.

permit extrapolation to greater spacing could not thus be realized. A different tack was hence followed in approximating the desired conditions of infinite spacing.

Since the profile of the standing eddy behind the test plate was roughly elliptical in form (figure 5), it was assumed that the streamlines at a considerable distance would approximate those of potential flow past either an ellipse or a Rankine oval of comparable dimensions. The latter being somewhat easier to describe mathematically, the location and strength of the source and sink were determined so as to yield a profile coinciding most closely with the measured eddy profile for $B/b = 12$. The resulting stream function had the form

$$\psi = -yu_0 + 0{\cdot}56\,bu_0\left[\tan^{-1}\frac{y}{x-3{\cdot}4\,b} - \tan^{-1}\frac{y}{x+3{\cdot}4\,b}\right],$$

the profile of the oval corresponding to $\psi = 0$. From this relationship it was possible to evaluate the coordinates of any desired streamline in the vicinity of the test-section wall; the maximum deviation of the streamline at the approximate lateral distance of 1·5 feet was found to be about 1·5 inches. Because of the boundary layer development along these walls, however, a still further change in their shape would be necessary to eliminate this additional wall effect. The displacement thickness of the boundary layer as a function of distance along this tunnel was hence evaluated by a combination of velocity measurement with extrapolation, in accordance with known boundary layer relationships; its entire variation was found to be slightly less than 1/4 inch over the 10-foot length of test section. False boundaries of hardboard were then introduced along the ceiling and floor of the test section; they were curved according to the stream function of the eddy profile, flared by an amount equal to the displacement thickness of the boundary layer, and smoothly joined to the bell entrance of the test section.

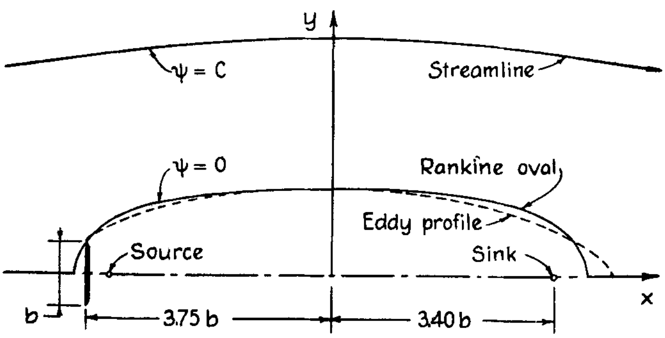

Figure 5. Derivation of stream function for Rankine oval having proportions of standing eddy.

With the falsework in position, the velocity distribution over successive vertical sections throughout the eddy region was again measured with the Pitot cylinder. In addition to such velocity traverses, measurements were also made of the pressure distribution at the same sections, and of the pressure over the faces of the test plate and tail plate. Pressure measurements at the plates themselves required simply the introduction of piezometer orifices along the one side, the velocity measurements being restricted to the same side in order to avoid the disturbing effect of the piezometer connections on the other. The pressure traverses in a plane normal to the two plates involved the use of a third plate, 1/8 inch thick, 5 inches wide (in the longitudinal direction), and about 18 inches high, sharply bevelled on the leading edge, and having a series of piezometer orifices on one side in a vertical row, which could be placed at any section. Finally, measurements were made with the constant-temperature type of hot-wire anemometer (Hubbard 1954) at the same vertical sections, the wires and circuit being so arranged as to yield the root-mean-square values of the three components of fluctuation and the cross product of the components in the plane normal to the two test plates.

Analysis of measured data

Had the revision in tunnel profile actually been sufficient to simulate infinite-fluid conditions, the experimental results would have yielded directly the patterns of velocity, pressure, and turbulence produced by flow over a normal wall. However, it will be recalled that the boundary profile was determined on the assumption of irrotational flow around a Rankine oval, whereas flow over a wall—though it may be essentially irrotational at the outset—departs from this state more and more with distance downstream, as a result of the pronounced shear and generation of turbulence at the border of the standing eddy. In other words, although a fairly successful effort had been made to compensate for the presence of the eddy and the development of the boundary layer on the tunnel walls, there remained uncompensated the effect of the wake of the plate itself, the importance of which was not at once appreciated. Finally, rather than modify the tunnel profile and repeat the measurements a second time, it was decided to determine by analytical means the magnitude of the corrections still required.

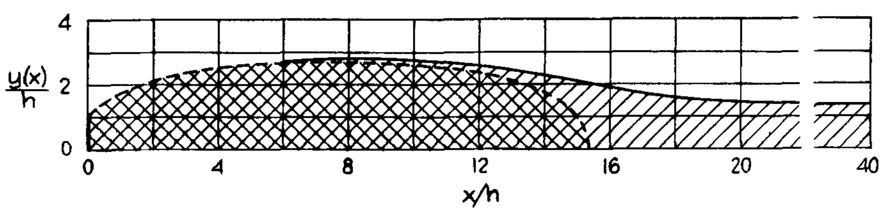

Figure 6. Displacement effect of standing eddy and turbulent wake.

With reference to figure 6 (in which the wall height h is introduced as reference length), the total effect of the wall upon the surrounding flow could be considered equivalent to that of the combination of the Rankine body (shown by double cross-hatching) with an additional tail section (shown by single cross-hatching) corresponding to the displacement thickness of the flow in the wake. The total thickness of the combination was assumed to be given to a first approximation by

$$\delta^* = \int_0^{\delta} \frac{u - u_{max}}{u_{max}} dy,$$

the quantity δ representing the value of y, just outside the wake, at which the velocity becomes a maximum.

Just as the profile of the eddy was simulated by the combination of a source and a sink with a uniform flow, the composite profile of the eddy and the tail section (or the profile of either one alone) could be realized by a distribution of sources and sinks, or, more conveniently, by a distribution of doublets (a method analogous to that of Landweber 1951), in place of the single source and sink of the Rankine body. For the case of an infinite fluid the stream function would then have the form

$$\psi = -yu_0 - \int_a^\infty \frac{m(t)\,y}{(x-t)^2+y^2}\,dt,$$

in which $m(t)$ is the doublet strength per unit length as a function of distance t along the line of symmetry and a is the abscissa of the initial point of the distribution which will yield the desired profile for the condition $\psi = 0$. For the case of flow that is confined between parallel walls (i.e., for a series of similar bodies) with the spacing B, on the other hand, the stream function would have to be written as

$$\psi = -yu_0 + \frac{\pi}{B}\int_a^\infty \frac{m(t)\sin 2\pi y/B}{\cosh 2\pi(x-t)/B - \cos 2\pi y/B}\,dt,$$

the doublets being distributed symmetrically along a series of parallel axes midway between the walls.

Because the tunnel boundaries in the present experiments were formed according to the approximate stream function of the eddy alone, the differences between the velocity distributions indicated by the proper space derivatives of these two equations must be due almost entirely to the tail section or wake. Evaluation of the difference, in the form of an approximate correction factor, hence depended upon determination of the doublet distribution $m(t)$ for the tail section alone. Fortunately, the elements shown in figure 6 were sufficiently slender for a considerable simplification (again by analogy with the method of Landweber (1951)) to be made. Thus, upon replacing $m(t)$ by $m(x)$ with $\psi = 0$ in the stream function for the infinite fluid, it could be shown that, with acceptable accuracy,

$$m(x) = -u_0 y(x)\left[\tfrac{1}{2}\pi + \tan^{-1}\frac{x-a}{y}\right]^{-1}$$

The computed distribution of $m(x)$ for the tail section (and as well for the composite profile, with a value of a that would yield a satisfactory approximation in the vicinity of the plate) was that shown in figure 7. Use of the values indicated by the full curve thus permitted calculation of the factor by which the measured velocity outside the wake should be reduced to correspond with that of the infinite case. (Use of the composite values, it should be noted, will lead to an approximate stream function for the infinite case.)

The magnitude of the resulting corrections may be judged from figure 8 *a*, in which are superposed the longitudinal components of the measured velocities (the points themselves) and the corrected values (the distribution curves) at every other measured section. Also shown are the streamlines determined from the corrected distribution curves in the region of the eddy. From these it will be seen that it was actually of little moment whether or not the correction (based upon the assumption of irrotational flow) was made in the rotational region, where the velocity was already quite low. As a matter of fact, the velocity correction was so small in all zones as to have almost no effect upon the flow pattern itself. Its effect upon the pressure distribution, on the other hand, was considerable.

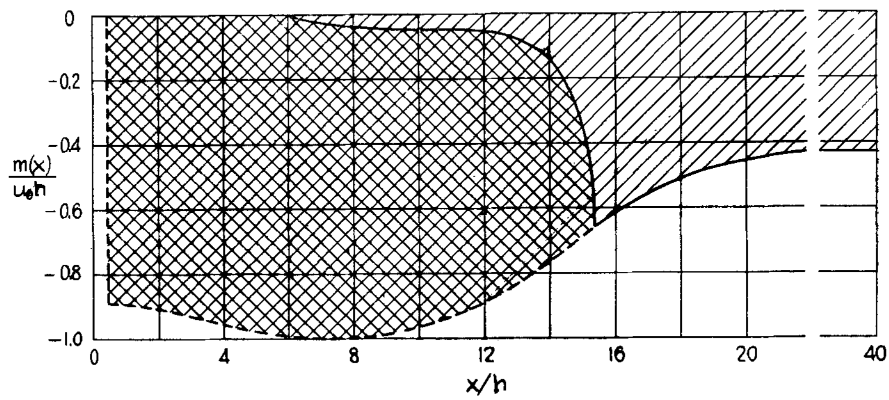

Figure 7. Distribution of doublet strength for eddy and wake displacement of figure 6.

Figure 8 *b* reproduces the actual measurements of the pressure intensity as points and the corrected values as continuous curves plotted to the left of the respective sections (i.e., negatively). Once again the correction was strictly applicable only outside the wake, for it was based upon the assumption of irrotational flow and constancy of the sum $\frac{1}{2}\rho(u^2+v^2)+p$. Unlike the velocity correction, however, that of the pressure could by no means be ignored within the wake, since there it was evidently a maximum. For want of a more precise method of estimating it, the customary boundary layer assumption of negligible pressure change over the distance δ had to be made; in other words, the correction at any point in the eddy was assumed to be the same as that just outside at the same cross-section. That the probable error in this assumption was small may be concluded from the fact that the measurements themselves varied only slightly across the eddy.

With the corrected magnitudes of the velocity and pressure at hand throughout the zone in question, it was a simple matter to compute and plot the distribution of total pressure (i.e., as would be read at the stagnation opening of a Pitot tube) relative to that in the ambient fluid, which is also shown in figure 8 *b*. Comparison of this sequence of curves with the locus of points of maximum velocity will indicate that the latter corresponds rather closely to the approximate limit of the zone of irrotational flow.

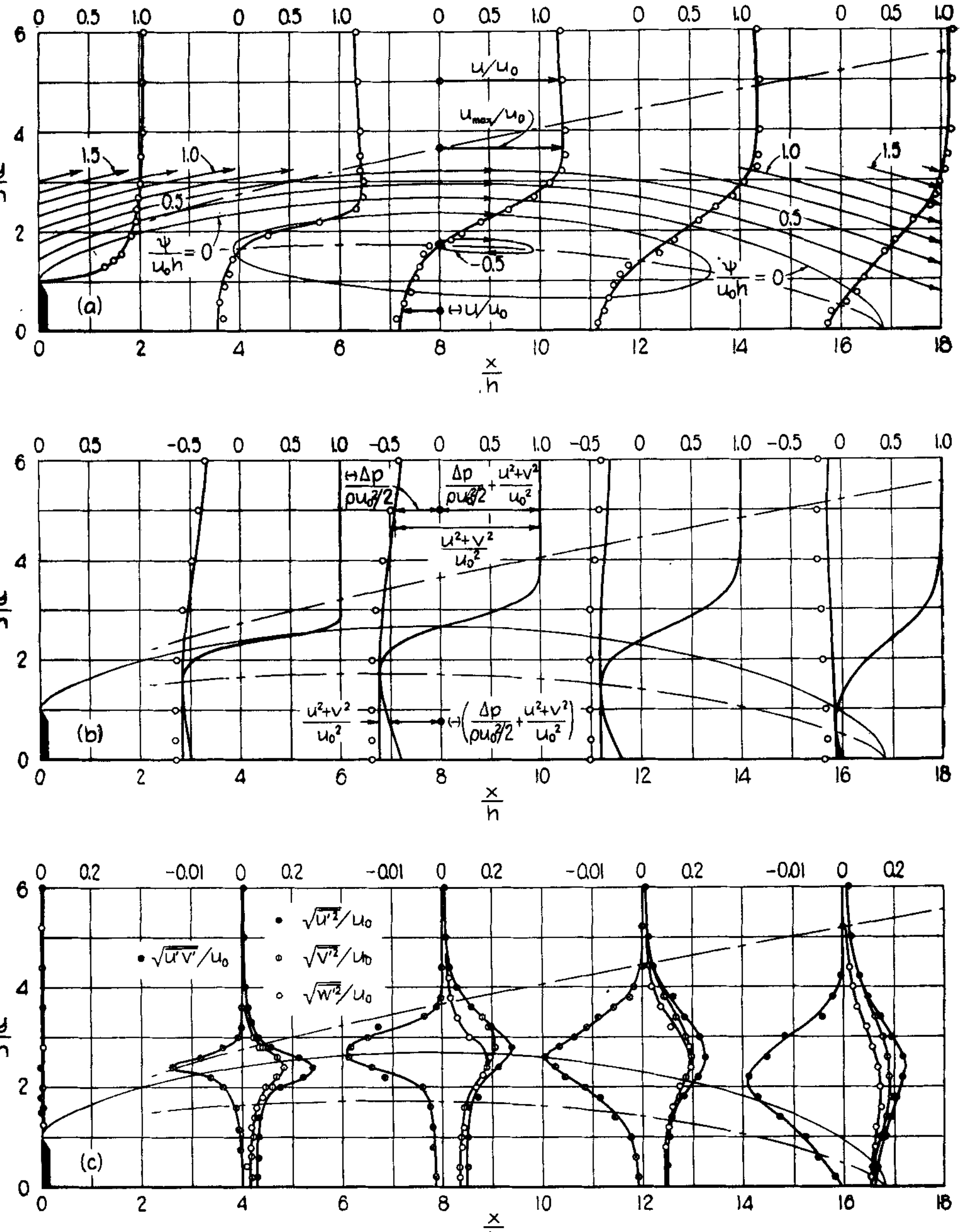

Figure 8. Experimental data and corrected distributions of (*a*) velocity, (*b*) pressure, and (*c*) turbulence, behind test plate.

The summary plots in figure 8*c* of the three components of the turbulent velocity and of the cross product of the two components in the plane of motion (obviously uncorrected for the wake effect) quite logically show the region of maximum turbulence and maximum shear to coincide at the outset with the mean streamline dividing the main flow from that in the zone of separation, remaining at about twice the wall height as the eddy terminates. Were data for additional characteristics of the turbulence at hand—in particular, the microscale—it would be readily possible to trace the transformation of the energy of the primary or mean flow through its secondary or turbulent stage to its final dissipation as heat (Hsu 1950). For the present, it must suffice to note that in the zone of maximum turbulence level the energy of the secondary motion is some 15% of that of the primary

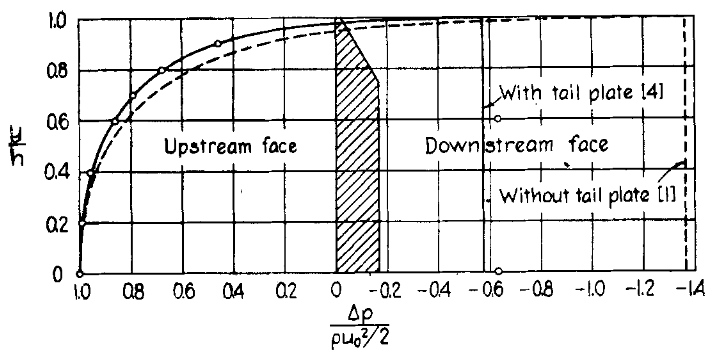

Figure 9. Distribution of pressure on test plate, with and without tail plate.

There remains to be considered here only the evaluation of the force F exerted by the flow upon the wall. From the direct measurements plotted in figure 9, the integral of the variable intensity on the front and the corrected constant intensity on the rear yields for the drag coefficient the magnitude

$$C_D = \frac{F/hL}{\frac{1}{2}\rho u_0^2} = 1{\cdot}38.$$

That this value is in accord with the change in flow pattern that the plate produced was demonstrated by evaluating the momentum equation

$$\int_0^\infty \left(1-\frac{u^2}{u_0^2}\right) d\left(\frac{y}{h}\right) + \int_0^\infty \frac{p_0-p}{\rho u_0^2}\, d\left(\frac{y}{h}\right) - \int_0^\infty \frac{\overline{u'^2}}{u_0^2}\, d\left(\frac{y}{h}\right) + \\ + \frac{\nu}{u_0 h}\int_0^\infty \frac{\partial(u/u_0)}{\partial(x/h)}\, d\left(\frac{y}{h}\right) = \frac{F/hL}{\rho u_0^2} = \tfrac{1}{2}C_D$$

between a section far upstream (i.e. where $u/u_0 = 1$) and an arbitrary section downstream. For the latter the section $x/h = 4$ was chosen, and the integration was carried out to—in lieu of infinity— the limit $y/h = 100$ (extrapolation beyond the zone of measurement being facilitated by use of the stream function based upon the distribution of $m(t)$ for the composite profile plotted in figure 7). The successive integrals were found to have the values $-2{\cdot}29$, $3{\cdot}04$, $0{\cdot}05$, and $0{\cdot}00$, respectively, with the result that $C_D = 1{\cdot}40$, which, as a residual value, is in unexpectedly good agreement with $1{\cdot}38$.

Interpretation of results

One must conclude from this investigation that the use of measurements made on isolated cylindrical bodies to approximate conditions in which the wake is not free to oscillate can result in rather large errors. The actual drag coefficient of the wall ($C_D = 1{\cdot}4$), for example, is only two-thirds that of the isolated plate ($C_D = 2{\cdot}1$), and the relative pressure change $\Delta p/\frac{1}{2}\rho_0^2$ behind the wall ($-0{\cdot}57$) is less than half the mean value ($-1{\cdot}36$) behind the plate. That this corresponds to a considerable difference in the length of the mean eddy zone (in accordance with the Riabouchinsky and related theories (Birkhoff 1950) that the free-streamline curvature must increase to produce a pressure decrease in a wake or cavity) is verified by the comparative measurements made in the course of the present study, the standing eddy behind the wall being more than three times as long ($l/h = 17$) as the homologous portion of the temporal mean pattern of flow past an isolated plate ($l/b = 2{\cdot}3$).

Such reduction in drag and pressure drop, and such lengthening of the average eddy zone, point to the fact that a major difference between the two cases lies in the considerable amount of mean-flow energy that is required to produce the vortices that are shed alternately into the oscillating wake. At low Reynolds numbers these vortices are known to persist for a considerable distance prior to their disintegration into turbulence; in such circumstances, the wake behind the isolated plate should be narrower than that behind the wall. At high Reynolds numbers, on the other hand, the vortices tend to disintegrate quite rapidly; the rate of lateral spread of the turbulent zone behind the plate then probably exceeds that behind the wall. Since at all Reynolds numbers beyond a few thousand the standing eddy behind the wall appears to maintain an essentially constant form, the same can reasonably be expected of its wake as a whole; and since both the scale and the energy of the turbulence generated at the periphery of the standing eddy are smaller than the corresponding quantities in the oscillating wake, the disturbance caused by the wall should invariably dissipate more rapidly.

How great an effect the existence of boundary shear upstream from the wall (deliberately set aside at the outset because of its undefined magnitude) would actually have upon these characteristics can only be surmised. The complexity of its experimental evaluation is seen from the fact that under even the simplest probable conditions of velocity variation—for example, the semi-logarithmic distribution $u = A+B\log y$—at least two variables must be considered: in brief, a magnitude and a rate of change. Fortunately, a phenomenon analogous to that under discussion is found in flow through a pipe orifice, for which it is both logical and experimentally demonstrable that maintaining a constant mean velocity and varying the transverse distribution by roughening the pipe surface will reduce both the pressure change and the jet contraction. There is little basic difference between the axisymmetric orifice and the two-dimensional slot, and the slot differs even less from the boundary arrangement investigated in the present study. One may therefore conclude that an increase in the velocity variation over a given vertical distance (say $2h$, the velocity at level h thereby remaining constant) could be expected to cause a decrease in the drag of a wall, in the pressure reduction behind it, and in the size of the standing eddy which it produces. Until the extent of this effect has been determined experimentally, however, the prediction of such related phenomena as cavitation in the eddies formed by boundary projections must continue according to the rule of thumb that the significant velocity is that at a boundary distance equal to the height of the projection.

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